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# Proof Of Fermat's Last Theorem By Choosing Two Unknowns in the Integer Solution Are Prime Exponents

Dr.k sreedevi, Dr.braou, Mr. Thiruchinarpalli Srinivas, Dr.Braou

Dept. of Mathematics, Hyderabad, Telangana, India
*Correspondence:sri.du.1980@gmail.com

In this paper we are revisits well known problem in number theory ' proof of Fermat's last theorem ' with different perspective .Also we are presented for n>2, Diophantine equations K($x^n + y^n$) = $z^n$ and $x^n + y^n = L\, z^n$ are satisfied by some positive prime exponents of x, y, z with some sufficient values of K and L. But it is not possible to find positive integers x, y and z, which are satisfies above equations with exactly K=1 and L=1. Clearly it proves the Fermat's last theorem, which states that No positive integers of x, y, z are satisfies the equation $x^n + y^n = z^n$ for n > 2.
**Keywords**: Fermat's Last theorem, Diophantine equation, Prime Exponents.

**Introduction**

We know that every integer is either prime or product of primes. Also we can verify easily above equations K($x^n + y^n$) = $z^n$ and $x^n + y^n = L\, z^n$ are satisfied by some positive integers x, y, z
(which are primes or product of primes with exponent power is 1) with some sufficient values of K and L are not equal to 1 for n > 2. i.e we can verify Fermat's last theorem by choosing of x, y, z (exponent power is 1) to solve for K and L. Some examples are represented in below table.

TABLE 1:

| Choose n value | Choose x value | Choose y value | Choose z value | K $=\frac{z^n}{x^n+y^n}$ | L $=\frac{x^n+y^n}{z^n}$ |
|---|---|---|---|---|---|
| 3 | 2 | 3 | 5 | 3.57 | 0.28 |
| 3 | 3 | 4 | 5 | 1.37 | 0.728 |
| 3 | 2 | 5 | 7 | 2.57 | 0.3877 |
| 3 | 3 | 5 | 7 | 2.2565 | 0.4431 |
| 4 | 3 | 5 | 11 | 8.7565 | 0.1141 |
| 4 | 3 | 6 | 7 | 1.41152 | 0.7084 |
| 4 | 5 | 4 | 6 | 1.1428 | 0.875 |

Now we can solve for the values of K and L by choosing x and y are prime exponents whose power is more than one for proving Fermat's Last theorem.

**Working rule:**

Consider the Diophantine equations K($x^n + y^n$) = $z^n$ and $x^n + y^n = L\, z^n$. we are worked for finding 'z', 'K', 'L' values by choosing of x and y are prime exponents of 2,3 and 5.

Case 1: x, y is represented by Exponent of 2

Theorem 1: Let x=$2^p$ , $y = 2^q$ , $z = 2^p\left(1 + 2^{n(q-p)}\right), K = \left(1 + 2^{n(q-p)}\right)^{n-1}$ are satisfies the equation K($x^n + y^n$)= $z^n$ for all integer values of p ≥1 ,q $\geq 1, p < q$ , n≥1.

Proof: Let x= $2^p, y = 2^q$

Consider $x^n + y^n = (2^p)^n + (2^q)^n$

$x^n + y^n = 2^{np} + 2^{nq}$

$x^n + y^n = 2^{np}\left(1 + 2^{n(q-p)}\right)$

Now we can multiply both side with $\left(1 + 2^{n(q-p)}\right)^{n-1}$ ,we obtain that

$\left(1 + 2^{n(q-p)}\right)^{n-1} (x^n + y^n) = 2^{np}\left(1 + 2^{n(q-p)}\right)^n$

$\left(1 + 2^{n(q-p)}\right)^{n-1} (x^n + y^n) = \left(2^p\left(1 + 2^{n(q-p)}\right)\right)^n$

Without loss generality , we can assume that K= $\left(1 + 2^{n(q-p)}\right)^{n-1}$ and z= $2^p\left(1 + 2^{n(q-p}\right)$

Then above equation is reduced as K($x^n + y^n$)= $z^n$. we can easily verify the Proof of Fermat's Last theorem by substitute the values of p, q and n to solve for K value (It must be not equal to one, for all values of p, q and n.)

Lemma 1: Without loss of generality, from above theorem replace q = p +1,

Let x= $2^p$ , $y = 2^{p+1}$ , $z = 2^p(1 + 2^n), K = (1 + 2^n)^{n-1}$ are satisfies the equation

K($x^n + y^n$)= $z^n$ for all integar values of p≥1 ,n≥1.

Proof: Let x= $2^p, y = 2^{p+1}$

Consider $x^n + y^n = (2^p)^n + (2^{p+1})^n$

$x^n + y^n = 2^{np} + 2^{np+n}$

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$x^n + y^n = 2^{np}(1 + 2^n)$

Now we can multiply both side with $(1 + 2^n)^{n-1}$ ,we obtain that

$(1 + 2^n)^{n-1}(x^n + y^n) = 2^{np}(1 + 2^n)^n$

$(1 + 2^n)^{n-1}(x^n + y^n) = \left(2^p(1 + 2^n)\right)^n$

Without loss generality , we can assume that K= $(1 + 2^n)^{n-1}$ and z= $2^p(1 + 2^n)$

Then above equation is reduced as K$(x^n + y^n)$= $z^n$.

TABLE 2: We can verify the triplets (x, y, z) are satisfies above equation by taking some values of p & n

| n | p | X = $2^p$ | Y= $2^{p+1}$ | Z= $2^p(1+2^n)$ | K= $(1+2^n)^{n-1}$ | K( $x^n + y^n$) | $z^n$ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 4 | 6 | 1 | 6 | 6 |
| 2 | 1 | 2 | 4 | 10 | 5 | 100 | 100 |
| 2 | 2 | 4 | 8 | 20 | 5 | 400 | 400 |
| 3 | 1 | 2 | 4 | 18 | 81 | 5832 | 5832 |
| 3 | 2 | 4 | 8 | 36 | 81 | 46656 | 46656 |
| 4 | 1 | 2 | 4 | 34 | 4913 | 1336336 | 1336336 |
| 5 | 2 | 4 | 8 | 132 | 1185921 | 40074642432 | 40074642432 |
| 6 | 2 | 4 | 8 | 260 | 1073741825 | 30891577600000 | 30891577600000 |

Clearly K=1, only for n=1. And all other cases K is more than 1. It follows that Fermat's last theorem is verified for " No positive integers x, y, z are satisfies the equation $x^n + y^n = z^n$ for any integer n > 2.

THEOREM 2: Let x=$2^p$ , $y = 2^q$ , $z = 2^p$, $L = 1 + 2^{n(q-p)}$ are satisfies the equation

$x^n + y^n = L\,z^n$ for all integer values of p ≥1 , q $\geq 1$, n≥1.

Proof: Let x= $2^p$, $y = 2^q$

Consider $x^n + y^n$ = $(2^p)^n$+$(2^q)^n$

$x^n + y^n = 2^{np} + 2^{nq}$

$x^n + y^n = 2^{np}\left(1 + 2^{n(q-p)}\right)$

Without loss generality, we can assume that L=$1 + 2^{n(q-p)}$ and z= $2^p$

Then above equation is reduced as $x^n + y^n = L\,z^n$. we can easily verify the Proof of Fermat's Last theorem by substitute the values of p, q and n to solve for L value (It must be not equal to one, for all values of p, q and n.)

Lemma 2: From above theorem, without loss of generality replace q = p +1

Let x=$2^p$ , $y = 2^{p+1}$ , $z = 2^p$, $L = 1 + 2^n$ are satisfies the equation

$x^n + y^n = L\,z^n$ for all integar values of p≥1 ,n≥1.

Proof: Let x= $2^p$, $y = 2^{p+1}$

Consider $x^n + y^n$ = $(2^p)^n$+$(2^{p+1})^n$

$x^n + y^n = 2^{np} + 2^{np+n}$

$x^n + y^n = 2^{np}(1 + 2^n)$

Without loss generality, we can assume that L=$1 + 2^n$ and z= $2^p$

Then above equation is reduced as $x^n + y^n = L\,z^n$

TABLE 3: We can verify the triplets (x, y, z) are satisfies above equation by taking some values of p & n

| n | p | X= $2^p$ | Y= $2^{p+1}$ | Z= $2^p$ | $L = 1 + 2^n$ | $x^n + y^n$ | L$z^n$ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 4 | 2 | 3 | 6 | 6 |
| 2 | 1 | 2 | 4 | 2 | 5 | 20 | 20 |
| 2 | 2 | 4 | 8 | 4 | 5 | 80 | 80 |
| 3 | 1 | 2 | 4 | 2 | 9 | 72 | 72 |
| 3 | 2 | 4 | 8 | 4 | 9 | 576 | 576 |
| 4 | 2 | 4 | 8 | 4 | 17 | 4352 | 4352 |
| 5 | 2 | 4 | 8 | 4 | 33 | 33792 | 33792 |
| 6 | 2 | 4 | 8 | 4 | 65 | 266240 | 266240 |
| 7 | 2 | 4 | 8 | 4 | 129 | 2113536 | 2113536 |
| 8 | 2 | 4 | 8 | 4 | 257 | 16842752 | 16842752 |
| 9 | 2 | 4 | 8 | 4 | 513 | 134479872 | 134479872 |

Clearly L is greater than 1. It follows that Fermat's last theorem is verified for " No positive integers x, y ,z are satisfies the equation $x^n + y^n = z^n$ for any integar n > 2.

Case 2: x, y are represented by Exponent of 3

Theorem 3: Let x=$3^p$ , $y = 3^q$ , $z = 3^p\left(1 + 3^{n(q-p)}\right)$, $K = \left(1 + 3^{n(q-p)}\right)^{n-1}$ are satisfies the equation K$(x^n + y^n)$= $z^n$ for all integer values of p ≥1 , q$> p$, n≥1.

Proof: Let x= $3^p$, $y = 3^q$

Consider $x^n + y^n$ = $(3^p)^n$+$(3^q)^n$

$x^n + y^n = 3^{np} + 3^{nq}$

$x^n + y^n = 3^{np}\left(1 + 3^{n(q-p)}\right)$

Now we can multiply both side with $\left(1+3^{n(q-p)}\right)^{n-1}$ ,we obtain that

$$\left(1+3^{n(q-p)}\right)^{n-1}\ (x^n+y^n) = 3^{np}\left(1+3^{n(q-p)}\right)^{n}$$

$$\left(1+3^{n(q-p)}\right)^{n-1}\ (x^n+y^n) = \left(3^{p}\left(1+3^{n(q-p)}\right)\right)^{n}$$

Without loss generality, we can assume that K= $\left(1+3^{n(q-p)}\right)^{n-1}$ and z= $3^p\left(1+3^{n(q-p)}\right)$

Then above equation is reduced as K($x^n+y^n$)= $z^n$. we can easily verify the Proof of Fermat's Last theorem by substitute the values of p, q and n to solve for K value (It must be not equal to one, for all values of p, q and n.)

Lemma 3: Without Loss of generality, from above theorem replace q = p +1,

Let x= $3^p$ , $y = 3^{p+1}$ , $z = 3^p(1+3^n), K = (1+3^n)^{n-1}$ are satisfies the equation

K($x^n+y^n$)= $z^n$ for all integar values of p≥1 ,n≥1.

Proof: Let x= $3^p, y = 3^{p+1}$

Consider $x^n+y^n = (3^p)^n+(3^{p+1})^n$

$$x^n+y^n = 3^{np}+3^{np+n}$$

$$x^n+y^n = 3^{np}\ (1+3^n)$$

Now we can multiply both side with $(1+3^n)^{n-1}$ ,we obtain that

$$(1+3^n)^{n-1}\ (x^n+y^n) = 3^{np}(1+3^n)^n$$

$$(1+3^n)^{n-1}\ (x^n+y^n) = \left(3^p(1+3^n)\right)^n$$

Without loss generality, we can assume that K= $(1+3^n)^{n-1}$ and z= $3^p(1+3^n)$

Then above equation is reduced as K($x^n+y^n$)= $z^n$

TABLE 4: We can verify the triplets (x, y, z) are satisfies above equation by taking some values of p & n

| n | p | X= $3^p$ | Y= $3^{p+1}$ | Z= $3^p(1+3^n)$ | K= $(1+3^n)^{n-}$ | K( $x^n+y^n$) | $z^n$ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 3 | 9 | 12 | 1 | 12 | 12 |
| 2 | 1 | 3 | 9 | 30 | 10 | 900 | 900 |
| 2 | 2 | 9 | 27 | 90 | 10 | 8100 | 8100 |
| 3 | 1 | 3 | 9 | 84 | 784 | 592704 | 592704 |
| 3 | 2 | 9 | 27 | 252 | 784 | 16003008 | 16003008 |
| 4 | 1 | 3 | 9 | 246 | 551368 | 3662186256 | 3662186256 |
| 4 | 2 | 9 | 27 | 738 | 551368 | 296637086736 | 296637086736 |
| 4 | 3 | 27 | 81 | 2214 | 551368 | 24027604025616 | 24027604025616 |

Clearly K=1, only for n=1. And all other cases K is more than 1. It follows that Fermat's last theorem is verified for " No positive integers x, y, z are satisfies the equation $x^n+y^n = z^n$ for any integar n > 2.

Theorem 4: Let x=$3^p$ , $y = 3^q$ , $z = 3^p$, $L = 1+3^{n(q-p)}$ are satisfies the equation

$x^n+y^n = L\,z^n$ for all integer values of p ≥1 , q> $p$ ,n ≥1.

Proof: Let x= $3^p$, $y = 3^q$

Consider $x^n+y^n = (3^p)^n+(3^q)^n$

$$x^n+y^n = 3^{np}+3^{nq}$$

$$x^n+y^n = 3^{np}\left(1+3^{n(q-p)}\right)$$

Without loss generality, we can assume that L=$1+3^{n(q-p)}$ and z= $3^p$

Then above equation is reduced as $x^n+y^n = L\,z^n$. we can easily verify the Proof of Fermat's Last theorem by substitute the values of p, q and n to solve for L value (It must be not equal to one , for all values of p, q and n.)

Lemma 4: Without loss of generality replace q = p +1,

Let x=$3^p$ , $y = 3^{p+1}$ , $z = 3^p$, $L = 1+3^n$ are satisfies the equation

$x^n+y^n = L\,z^n$ for all integer values of p ≥1 , n ≥1.

Now we can go to prove that $x^n+y^n = L\,z^n$ for all integer values of p ≥1 , n ≥1.

Proof: Let x= $3^p$, $y = 3^{p+1}$

Consider $x^n+y^n = (3^p)^n+(3^{p+1})^n$

$$x^n+y^n = 3^{np}+3^{np+n}$$

$$x^n+y^n = 3^{np}\ (1+3^n)$$

Without loss generality, we can assume that L=$1+3^n$ and z= $3^p$

Then above equation is reduced as $x^n+y^n = L\,z^n$

TABLE 5: We can verify the triplets (x ,y ,z) are satisfies above equation by taking some values of p & n

| n | p | X= $3^p$ | Y= $3^{p+1}$ | Z= $3^p$ | $L = 1+3^n$ | $x^n+y^n$ | L$z^n$ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 3 | 9 | 3 | 4 | 12 | 12 |
| 2 | 1 | 3 | 9 | 3 | 10 | 90 | 90 |
| 2 | 2 | 9 | 27 | 9 | 10 | 810 | 810 |
| 3 | 1 | 3 | 9 | 3 | 28 | 756 | 756 |
| 3 | 2 | 9 | 27 | 9 | 28 | 20412 | 20412 |
| 4 | 2 | 9 | 27 | 9 | 82 | 538002 | 538002 |
| 4 | 3 | 27 | 81 | 27 | 82 | 43578162 | 43578162 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 4 | 4 | 81 | 243 | 81 | 82 | 3529831122 | 3529831122 |
| 5 | 1 | 3 | 9 | 3 | 244 | 59292 | 59292 |
| 5 | 2 | 9 | 27 | 9 | 244 | 14407956 | 14407956 |
| 5 | 3 | 27 | 81 | 27 | 244 | 3501133308 | 3501133308 |
| 6 | 1 | 3 | 9 | 3 | 730 | 532170 | 532170 |

Clearly L is greater than 1. It follows that Fermat's last theorem is verified for " No positive integers x, y, z are satisfies the equation $x^n + y^n = z^n$ for any integar n > 2.

Case 3: x, y is represented by Exponent of 5

Theorem 5: Let x= $5^p$ , $y = 5^q$ , $z = 5^p\left(1 + 5^{n(q-p)}\right)$, $K = \left(1 + 5^{n(q-p)}\right)^{n-1}$ are satisfies the equation K($x^n + y^n$)= $z^n$ for all integer values of p ≥1 , q > p ,n ≥1.

Proof: Let x= $5^p$, $y = 5^q$

Consider $x^n + y^n = (5^p)^n + (5^q)^n$

$$x^n + y^n = 5^{np} + 5^{nq}$$

$$x^n + y^n = 5^{np}\left(1 + 5^{n(q-p)}\right)$$

Now we can multiply both side with $\left(1 + 5^{n(q-p)}\right)^{n-1}$ ,we obtain that

$$\left(1 + 5^{n(q-p)}\right)^{n-1} (x^n + y^n) = 5^{np}\left(1 + 5^{n(q-p)}\right)^n$$

$$\left(1 + 5^{n(q-p)}\right)^{n-1} (x^n + y^n) = \left(5^p\left(1 + 5^{n(q-p)}\right)\right)^n$$

Without loss generality, we can assume that K= $\left(1 + 5^{n(q-p)}\right)^{n-1}$ and z= $5^p\left(1 + 5^{n(q-p)}\right)$

Then above equation is reduced as K($x^n + y^n$)= $z^n$. we can easily verify the Proof of Fermat's Last theorem by substitute the values of p, q and n to solve for K value (It must be not equal to one, for all values of p, q and n.)

Lemma 5: Without loss of generality replace q = p +1,

Let x= $5^p$ , $y = 5^{p+1}$ , $z = 5^p(1 + 5^n)$, $K = (1 + 5^n)^{n-1}$ are satisfies the equation K($x^n + y^n$)= $z^n$ for all integer values of p ≥1 ,n ≥1.

Proof: Let x= $5^p$, $y = 5^{p+1}$

Consider $x^n + y^n = (5^p)^n + (5^{p+1})^n$

$$x^n + y^n = 5^{np} + 5^{np+n}$$

$$x^n + y^n = 5^{np}(1 + 5^n)$$

Now we can multiply both side with $(1 + 5^n)^{n-1}$ ,we obtain that

$$(1 + 5^n)^{n-1} (x^n + y^n) = 5^{np}(1 + 5^n)^n$$

$$(1 + 5^n)^{n-1} (x^n + y^n) = \left(5^p(1 + 5^n)\right)^n$$

Without loss generality, we can assume that K= $(1 + 5^n)^{n-1}$ and z= $5^p(1 + 5^n)$

Then above equation is reduced as K($x^n + y^n$)= $z^n$

TABLE 6: We can verify the triplets (x, y, z) are satisfies above equation by taking some values of p & n

| n | p | X= $5^p$ | Y= $5^{p+1}$ | Z= $5^p(1+5^n)$ | K= $(1+5^n)^{n-1}$ | K( $x^n + y^n$) | $z^n$ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 5 | 25 | 30 | 1 | 30 | 30 |
| 2 | 1 | 5 | 25 | 130 | 26 | 16900 | 16900 |
| 2 | 2 | 25 | 125 | 650 | 26 | 422500 | 422500 |
| 3 | 1 | 5 | 25 | 630 | 1587 | 250047000 | 250047000 |
| 3 | 2 | 25 | 125 | 3150 | 1587 | 31255875000 | 31255875000 |
| 4 | 1 | 5 | 25 | 3130 | 24531437 | 95979249610000 | 95979249610000 |

Clearly K=1, only for n=1. And all other cases K is more than 1. It follows that Fermat's last theorem is verified for " No positive integers x, y, z are satisfies the equation $x^n + y^n = z^n$ for any integar n > 2.

Theorem 6: Let x=$5^p$ , $y = 5^q$ , $z = 5^p, L = 1 + 5^{n(q-p)}$ are satisfies the equation

$x^n + y^n = L\, z^n$ for all integer values of p ≥1 ,q>p , n ≥ 1.

Proof: Let x= $5^p, y = 5^q$

Consider $x^n + y^n = (5^p)^n + (5^q)^n$

$$x^n + y^n = 5^{np} + 5^{nq}$$

$$x^n + y^n = 5^{np}\left(1 + 5^{n(q-p)}\right)$$

Without loss generality, we can assume that L=1 + $5^{n(q-p)}$ and z= $5^p$

Then above equation is reduced as $x^n + y^n = L\, z^n$. we can easily verify the Proof of Fermat's Last theorem by substitute the values of p, q and n to solve for L value (It must be not equal to one, for all values of p, q and n.)

Lemma 6: Without loss of generality from above theorem replace q = p +1

Let x=$5^p$ , $y = 5^{p+1}$ , $z = 5^p, L = 1 + 5^n$ are satisfies the equation

$x^n + y^n = L\, z^n$ for all integar values of p≥1 ,n≥1.

Proof: Let x= $5^p, y = 5^{p+1}$

Consider $x^n + y^n = (5^p)^n + (5^{p+1})^n$

$$x^n + y^n = 5^{np} + 5^{np+n}$$

$$x^n + y^n = 5^{np}(1 + 5^n)$$

Without loss generality, we can assume that L=1 + $5^n$ and z= $5^p$

Then above equation is reduced as $x^n + y^n = L\, z^n$

TABLE 7: We can verify the triplets(x, y, z) are satisfies above equation by taking some values of p & n

| n | p | X= $5^p$ | Y= $5^{p+1}$ | Z= $5^p$ | $L = 1 + 5^n$ | $x^n + y^n$ | $Lz^n$ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 5 | 25 | 5 | 6 | 30 | 30 |
| 2 | 1 | 5 | 25 | 5 | 26 | 650 | 650 |
| 2 | 2 | 25 | 125 | 25 | 26 | 16250 | 16250 |
| 3 | 1 | 5 | 25 | 5 | 126 | 15750 | 15750 |
| 3 | 2 | 25 | 125 | 25 | 126 | 1968750 | 1968750 |
| 4 | 1 | 5 | 25 | 5 | 626 | 391250 | 391250 |

Clearly L is greater than 1. It follows that Fermat's last theorem is verified for " No positive integers x, y ,z are satisfies the equation $x^n + y^n = z^n$ for any integer n > 2.

We can continue above procedure, with representing x and y in terms of different prime exponents of all integers and their corresponding arithmetical operations, we observed that in every case K and L are must be more than 1, It follows that for n>2 , It is not possible to find three positive integers x , y, z with K=1, L=1. In this way we can proved Fermat's Last theorem.

Conclusion In this paper we are presented for n>2, Diophantine equations K($x^n + y^n$) = $z^n$ and $x^n + y^n = L\, z^n$ are satisfied by some positive prime exponents of x, y, z with sufficient values of K and L. But it is not possible to find positive integers x, y and z, which are satisfies above equations with K=1 and L=1. Clearly it proves the Fermat's last theorem, which states that No positive integers of x, y, z are satisfies the equation $x^n + y^n = z^n$ for n > 2.

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