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Airy Function Solution of Two Classes of Nonlinear Evolution Equations

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Abstract: This paper studies the problem of constructing the new solutions of the mKdV equation and STO equation. By using the Airy equation and its solution, the new solutions of mKdV equation and STO equation are constructed. First of all, through function transformation, mKdV equation and STO equation can be turned into Airy function to solve. Then, construct the solution of mKdV equation and STO equation by using the solution of Airy function. Finally, the properties of these solutions are analyzed by means of the symbolic computing system Mathematica.

Key words: Function transformation; Airy function; mKdV equation; STO equation

I. Introduction

The study of nonlinear evolution equations is of great significance to many fields such as physics. The mKdV equation (1) is an important equation in nonlinear mathematical physics. It has been applied in many fields, especially in plasma physics. In-depth study on it will help to solve practical physical problems. The Sharma Tasso Olver (STO) equation (2) is also of great significance in mathematics and physics. In the field of physics, it can describe some real physical models, such as soliton fission and fusion. In recent years, many physicists and mathematicians have conducted in-depth research on it [4-10].

In reference [1-10], Riemann theta function, Riccati function, Jacobi elliptic function, hyperbolic function, trigonometric function, and other solutions of mKdV equation and STO equation [5] have been obtained, but Airy function solutions have not been obtained.

The two kinds of nonlinear evolution equations studied in this paper are as follows

$$u_t + \alpha u^2 u_x + \beta u_{xxx} = 0, \qquad (1)$$

$$u_t + \alpha u^2 u_x + \beta u_x^2 + \gamma u u_{xx} + u_{xxx} = 0, \qquad (2)$$

where α, β, γ are constants.

In this paper, the Airy function solutions of nonlinear evolution equations (1) and (2) are constructed by using function transformation method and auxiliary equation method, and the properties of the solutions are analyzed.

II. The solutions of airy function [11]

The airy equation considered is as follows:

$$y'' - kxy = 0, \qquad (3)$$

where $k \neq 0$. The general solution of airy equation is

$$y = c_1 Ai(x) + c_2 Bi(x), \qquad (4)$$

where c_1, c_2 are arbitrary constants.

III. Functional transformation

3.1 mKdV equation and function transformation

In order to obtain the solutions of Airy function, first assumes the function transformation

$$\begin{cases} u(x,t) = \xi_x \dfrac{d\ln(F(\xi))}{d\xi}, \\ \xi_{xx} = 0, \end{cases} \qquad (5)$$

where $\xi = \xi(x,t)$, $F(\xi)$ satisfies the equation (3), so it has the following relationship

$$
\begin{cases}
F'' = k\xi F, \\
F''' = k(F + \xi^2 F'), \\
F^{(4)} = k^2\xi^2 F + 2kF', \\
F^{(5)} = 4k^2\xi F + k^2\xi^2 F'.
\end{cases}
\tag{6}
$$

Substituting equations (5) and (6) into mKdV equation (1) and sorting out

$$
(-\alpha - 6\beta)(F')^4(\xi_x)^4 + F^2(F')^2(-\xi_t\xi_x + (k\alpha + 8k\beta)(\xi
$$

$$
F^4(k\xi\xi_t\xi_x - 2k^2\beta\xi^2(\xi_x)^4) + F^3F'(-2k\beta(\xi_x)^4 + \xi_{xt}) =
$$

(7)

Let the coefficient of the $(F')^i(\xi_x)^j F^m$ term be zero, and the following equations are obtained.

$$
\begin{cases}
\alpha + 6\beta = 0, \\
\xi_{xt} = 2k\beta\xi_x^4, \\
\xi_{xx} = 0,
\end{cases}
\tag{8}
$$

Solve equation group (8) to obtain the following solution

$$
\begin{cases}
\alpha = -6\beta, \\
\xi = ((-6k\beta t - 3c_0)^{-\frac{1}{3}})x + f(t) + c,
\end{cases}
\tag{9}
$$

where $f(t)$ is an arbitrary function about t, and c, c_0 are arbitrary constants.

Therefore, mKdV equation (1) has Airy function solution in the following form:

$$
\begin{cases}
u(x,t) = (-6k\beta t - 3c_0)^{-\frac{1}{3}} \dfrac{d\ln(F(\xi))}{d\xi}, \\
F(\xi) = c_1 Ai(\xi) + c_2 Bi(\xi), \\
\xi = ((-6k\beta t - 3c_0)^{-\frac{1}{3}})x + f(t) + c.
\end{cases}
\tag{10}
$$

3.2 STO equation and function transformation

Substituting equations (5) and (6) into STO equation (2) and sorting out

$$
\begin{aligned}
&(-6 - \alpha + \beta + 2\gamma)(F')^4(\xi_x)^4 + \\
&F^2(F')^2(-\xi_t\xi_x + (8k + k\alpha - 2k\beta - 2k\gamma)\xi(\xi_x)^4) \\
&+F^4(k\xi\xi_t\xi_x + (-2k^2 + k^2\beta)\xi^2(\xi_x)^4) + \\
&F^3F'((-2k + k\gamma)(\xi_x)^4 + \xi_{xt}) = 0,
\end{aligned}
$$

(11)

Let the coefficient of the $(F')^i(\xi_x)^j F^m$ term in equation (11) be zero, and the following equations are obtained.

$$
\begin{cases}
6 + \alpha - \beta + 2\gamma = 0, \\
\xi_{xt} = (-2k + k\gamma)\xi_x^4, \\
\xi_{xx} = 0,
\end{cases}
\tag{12}
$$

Solve equation group (12) to obtain the following solution

$$
\begin{cases}
(\alpha, \beta, \gamma)^T = k_1(-5,1,0)^T + k_2(-4,0,1)^T + (-6,0,0)^T, \\
\xi = (((6 - 3\gamma)kt - 3c_0)^{-\frac{1}{3}})x + f(t) + c,
\end{cases}
$$

(13)

where $f(t)$ is an arbitrary function about t, and c, c_0 are arbitrary constants.

Therefore, STO equation (2) has Airy function solution in the following form:

$$
\begin{cases}
u(x,t) = ((6 - 3\gamma)kt - 3c_0)^{-\frac{1}{3}} \dfrac{d\ln(F(\xi))}{d\xi}, \\
F(\xi) = c_1 Ai(\xi) + c_2 Bi(\xi), \\
\xi = (((6 - 3\gamma)kt - 3c_0)^{-\frac{1}{3}})x + f(t) + c.
\end{cases}
$$

(14)

IV.The solutions and properties of mKdV equation and sto equation

The solutions of equations (1) and (2) are obtained by sorting out equations (10) and (14) respectively.

$$
u_1 = (-6k\beta t - 3c_0)^{-\frac{1}{3}} \frac{c_1 Ai'(\xi) + c_2 Bi'(\xi)}{c_1 Ai(\xi) + c_2 Bi(\xi)},
$$

$$
u_2 = ((6 - 3\gamma)kt - 3c_0)^{-\frac{1}{3}} \frac{c_1 Ai'(\xi) + c_2 Bi'(\xi)}{c_1 Ai(\xi) + c_2 Bi(\xi)}.
$$

Using the symbolic computing system Mathematica, analyze the properties of the solution through the following figures.

Case 1 when $f(t)$ is the hyperbolic function, $f(t)$ is the rational function, and $f(t)$ is the logarithmic function, periodic solutions, torsion solutions, and peak soliton solutions are obtained.

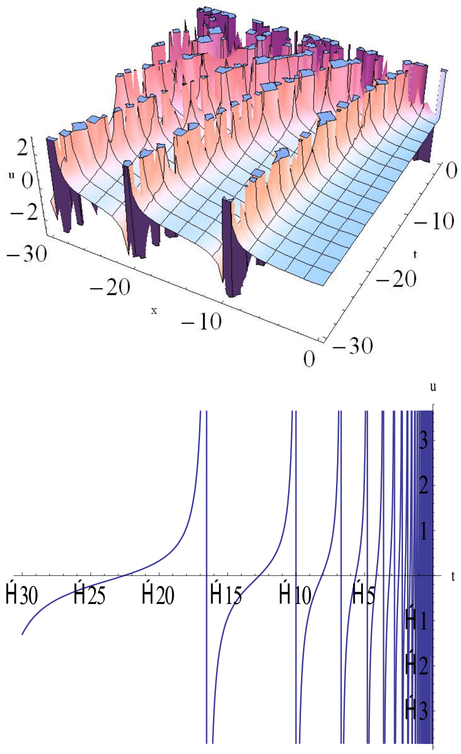

Fig.1 The solution of Airy function 1

Case 2 when $f(t)$ is a trigonometric function, periodic solutions, torsion solutions, bell solutions and peak soliton solutions are obtained.

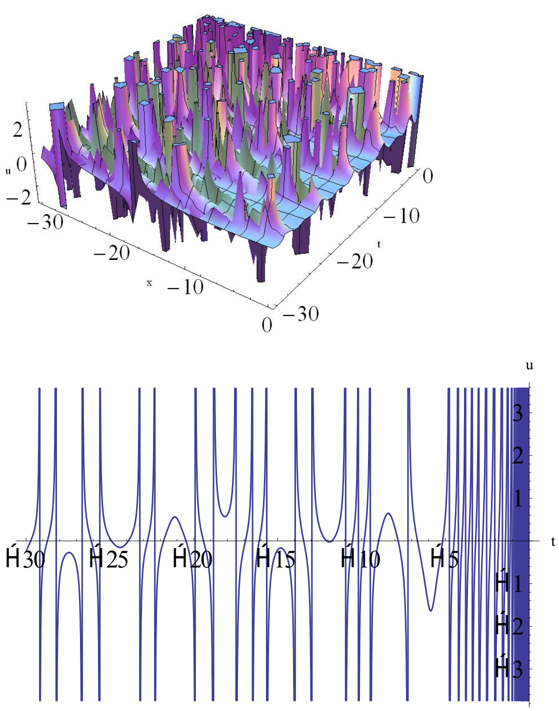

Fig.2 The solution of Airy function 2

Case 3 when $f(t)$ is a mixed function of trigonometric function and hyperbolic function, the periodic solution, kink solution, bell solution and peak soliton solution are obtained.

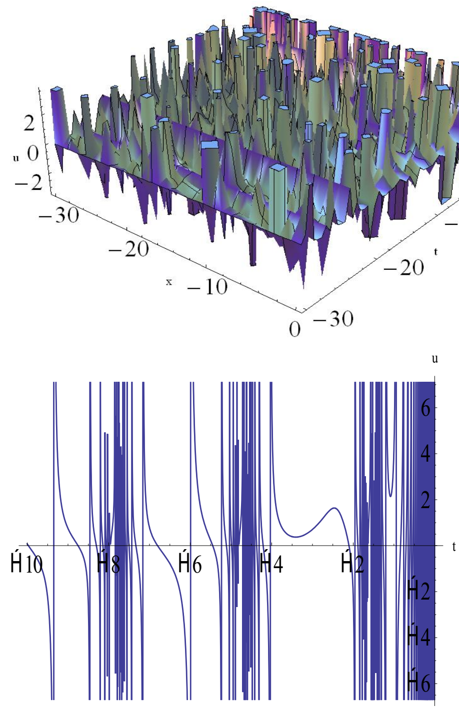

Fig.3 The solution of Airy function 3

It can be seen from the image that Airy function solution is a mixed form solution of many soliton solutions, and it is of great significance to obtain Airy function solution of nonlinear evolution equation.

5. Conclusion

To construct the solutions of nonlinear evolution equations, Riemann theta function, Jacobi elliptic function, hyperbolic function, trigonometric function and other solutions have been obtained, but few hyperfunctional solutions such as Airy function and Bessel function have been obtained. In this paper, through the assumption of formal solutions, the Airy function solutions of two kinds of nonlinear evolution equations are obtained by using the auxiliary function method and the homogeneous balance principle, and the symbolic computing system Mathematica is used to analyze the properties of the solutions. Periodic solutions, kink solutions, bell solutions and peak soliton solutions are obtained. The physical meaning of these solutions needs to be further studied.

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