Core Competency Oriented Problem-Solving Teaching
----Taking "Solving Application Problems with a System of Equations" as An Example

Li Xiaoling [1,2] ，Li Zhongxuan[3]

¹LinYi University，Shandong Province, Linyi 276000, China；
²Philippine Christian University Center for International Education，1004，Manila City，Republic of the Philippines；
³QingDao University，Shandong Province, Qingdao 266071, China.
Email: 823287917@qq.com, 1378881553@qq.com

Abstract: The teaching of Mathematical problem-solving is one of the three key teaching points in junior high school mathematics. Solving applied problems through equations (systems) is an important component of the first module of teaching content, "Numbers and Algebra", and holds a very important position. It is a key focus in junior high school mathematics problem-solving teaching. At present, under the premise of the implementation of the new curriculum reform in the field of mathematics in China, based on the recent publication of compulsory education curriculum standards,the teachers of mathematics should focus on cultivating excellent mathematical core literacy among junior high school students, and ensuring that they can independently cope with and solve problems in the math teaching of junior high school. Based on the above background, the author focuses on the newly published mathematics curriculum standards in 2022, and studies the teaching design of "solving application problems for systems of equations". In this paper, we explore the mathematical core literacy elements contained in the teaching of solving applied problems in equations, study the improvement path of mathematical problem-solving teaching, share the teaching design practice and thinking based on core literacy oriented problem-solving teaching, and the important role that this teaching design can play in cultivating students' core literacy .

Keywords: Problem-solving teaching, Core competencies,Equation system

Introduction

Mathematics teaching can generally be divided into three parts: teaching of mathematical concepts, teaching of mathematical propositions, and teaching of mathematical problem-solving[1]. Teaching mathematics problem-solving has become a fundamental teaching point in junior high school mathematics. The teaching content of junior high school mathematics includes four modules: numbers and algebra, graphics and geometry, statistics and probability, and synthesis and practice. Solving applied problems with equations (systems) is an important component of the first module "numbers and algebra", which plays a very important role and is a key focus in middle school mathematics problem-solving teaching.

The 2022 edition of the Mathematics Curriculum Standards for Compulsory Education proposes that core mathematical literacy mainly includes the following three aspects: (1) the ability to observe the real world from a mathematical perspective. In the compulsory education stage, the mathematical perspective mainly manifests as abstract ability, geometric intuition, spatial concepts, and innovative consciousness. (2) Can use mathematical thinking to think about the real world. In the compulsory education stage, mathematical thinking mainly manifests as: computational ability, reasoning consciousness, or reasoning ability. (3) Can express the real world in mathematical language. In the compulsory education stage, mathematical language mainly manifests as: data awareness or data concepts, model awareness or model concepts, and application awareness[2]. In the curriculum system of middle school mathematics, mathematics problem-solving teaching accounts for a relatively high proportion of class hours. Currently, middle school students still generally lack innovative practical thinking in mathematics problem-solving, leading to rigid mathematical thinking defects in the implementation of mathematical problem-solving operations. It can be seen that mathematics problem-solving teaching aims to promote the formation of better core literacy abilities among middle school students, So it is necessary for teachers to train middle school students to independently think and solve various types of mathematical problems. At present, under the premise of the implementation of the new curriculum reform in the field of mathematics in China, based on the recent publication of compulsory education curriculum standards, mathematics teachers should focus on cultivating excellent mathematical core literacy among middle school students, ensuring that they can independently cope with and solve problems in middle school mathematics teaching. Based on the above background, the author focuses on

the newly published mathematics curriculum standards in 2022. Taking the teaching design of the topic of "solving applied problems with equations" in the textbook of junior high school mathematics published by Qingdao Publishing House as an example, this article explores the mathematical core literacy elements contained in the teaching of solving applied problems with equations, and studies the improvement path of mathematical problem-solving teaching, Sharing the teaching design practice and thinking of problem-solving teaching based on core literacy orientation, as well as the important role that this teaching design can play in cultivating students' core literacy.

1 Content Analysis

1.1 Textbook Analysis
"Solving Application Problems with a System of Equations" is the content of Chapter 10, Section 4, Lesson 1 of the compulsory education textbook "Mathematics Grade 7 Volume 2" (2nd edition in 2012) published by Qingdao Publishing House [3]. It is about solving problems with a system of equations. The content of this lesson is to further learn how to solve practical problems such as travel problems and matching problems in daily life by constructing mathematical models and establishing equality relationships, based on students having mastered the application problems of solving univariate linear equations and solving binary linear equation systems. Through learning, students will further understand that equation systems can depict quantitative relationships in the real world. This mathematical model is also an important mathematical tool for solving practical problems. Solving application problems with binary linear equations is another important method after solving application problems with binary linear equations. It also lays the foundation for solving application problems with fractional equations. Therefore, it plays a bridging role and is an important part of the series of applications of equation thinking.

1.2 Analysis of learning situation
Students in the second semester of the first year of junior high school have a strong curiosity and thirst for knowledge. They also possess certain abilities in cooperation and exploration, as well as a strong sense of participation. Students already have the foundation to solve application problems by solving linear equations. They have a fundamental understanding of the steps to solve linear equations. What they need to do is to clarify the quantitative and isometric relationships in practical problems, list and solve linear equations, and further develop mathematical modeling ideas.

1.3 Teaching objectives
(1) Learn to use binary linear equations to solve various types of problems in daily life, such as itinerary, matching, and engineering coordination.

(2) Through the process of solving practical problems through a system of binary linear equations, and through activities such as practice and exploration, cultivate and improve the ability to move the eyes, mind, and hands, and develop core competencies of observing with a mathematical perspective, thinking with mathematical thinking, and expressing with mathematical language.

(3) Through practice and exploration activities, I pay more attention to life and have a deeper understanding of how mathematics comes from and serves life; Enhance the ability to explore, discover, and innovate; Cultivate a team spirit of unity and cooperation, a spirit of courage to explore and seek truth from facts.[3]

1.4 Key Points, Difficulties
The key points in the teaching process are those that run through the overall situation, drive comprehensiveness, and play a core role. It is determined by the position and role of the teaching content in the knowledge structure. It can be considered from several aspects: is it the core relative to the relevant parts of the textbook? Or consider whether it is the foundation for learning other content in the future? Or consider whether it has a wide range of applications? If one aspect receives a positive response, it can be concluded that it is the focus. In terms of the teaching content of this lesson, we can determine that the teaching focus should be on clarifying the unknown quantities, known quantities, and equal relationships in the problem, and listing a system of equations to solve application problems

Key points: Clarify the unknown quantities, known quantities, and equivalent relationships in the problem, and list a system of equations to solve the application problem.

The difficulties in the teaching process are those in understanding, mastering, or applying. Generally speaking, the content is relatively abstract, the structure is complex, the essential attributes are relatively hidden, new perspectives and methods need to be applied, or students lack necessary emotional understanding, which can be identified as difficulties. Therefore, the teaching difficulty of this lesson can be determined as: understanding the transformation thinking and mathematical model thinking of transforming real-life problems into mathematical problem-solving.

Difficulties: Understand the transformation thinking and mathematical model thinking of transforming real-life problems into mathematical problem-solving.

2 Teaching Design Practice Cases

2.1 Creating Situations and Introducing New Lessons

Teachers play the video, guide students to feel the speed of China, and follow the trend to introduce new lessons: today, let's follow these scenes to analyze the story behind these photos, let's walk into the Application Problem Solving of Equations, and learn to feel the charm of mathematics while feeling the speed of China's development.

Design intention: Using a camera lens as a medium, the entire lesson knowledge is integrated into one line, effectively concentrating students' attention.

Example question: In shot 1, Beijing Kexing and Changchun Biotechnology, two pharmaceutical companies, have a total of 50 workshops planned to be put into operation. Beijing Kexing has a production capacity of 300000 units per day in each workshop, while Changchun Biotechnology has a production capacity of 150000 units per day in each workshop. How many workshops should each have started production to ensure a total supply of 10.5 million units per day?

Follow up question: Can you solve this problem?

Follow up question: What are known quantities and unknown quantities? What are the equivalent relationships?

Joint activities between teachers and students： Review the methods for solving application problems with linear equations: (1)（审） Extract effective mathematical information from the problem stem, (review) clarify the problem, (2)（设） clarify known and unknown quantities, (assume) identify unknowns, (3)（列） (create)equations based on equal relationships, (4)（解） (solve)equations, (5)（验） (verify) whether the results match the actual problem meaning, (6)（答） provide (answer) solutions. The Chinese abbreviation is "审设列解验答".

2.2 Analogical exploration, forming ideas

Teacher's guidance: Can we simply summarize the steps as follows? (Reviewed and verified). Now quickly review the meaning of the problem, extract effective mathematical information, circle the known and unknown quantities on the study plan, and use the method of solving application problems using the univariate linear equation previously learned. (Project a classmate's answer and provide feedback)

Expansion: Carefully review the problem and its solving process, and think deeply about what type of problem it is? (Allocation issue). What is the core formula that appears in this type of problem? (Total amount = sum of all components) This is also the key to the problem-solving process: finding equal quantity relationships.

Question: Is there only one way to "set" an unknown quantity in the solution just now?

Joint activities between teachers and students： Students actively think and explore to come to conclusions: let the unknown quantity be either one or two. If one unknown quantity is set, it can be solved by solving a linear equation with one variable. If two unknown quantities are set, it can be solved by solving a linear equation with two variables.

Design intention: Infiltrate mathematical modeling ideas, clarify the starting point of problem-solving methods, and form analogies with solving application problems using univariate linear equations, laying a foundation for the learning of this lesson. In the analysis process of this example, the first intuition of students is that "solving application problems with equations" and "solving application problems with linear equations" are similar. Teachers should fully stimulate students' intuitive perception.

Exploration 1: If there are x workshops in Beijing Kexing and y workshops in Changchun Biotechnology that are put into operation, can you provide an equal quantity relationship equation (system of equations)?

Follow up question: Can you continue to solve this problem by analogy with the previous solution?

Student activity: Complete the problem-solving process using the method of solving applied problems using a system of equations.

Follow up question: What is the connection and difference between the two methods of answering?

Joint activities between teachers and students： Teachers and students explore together, students summarize and summarize.

Design intention: Starting from the recent development zone of knowledge formation, setting up valuable mathematical problems, inspiring students to actively think, guiding students to communicate and discuss, clarifying the research content of this section - establishing equations (systems), and solving practical problems through solving equations (systems), obtaining the research method of this chapter - equation (system) method, and comprehending the mathematical ideas of this chapter - mathematical modeling ideas, transformation ideas, equation ideas, Furthermore, in the midst of lively discussions and urgent expectations from students, new lessons are naturally introduced. By reviewing the research methods of solving application problems with linear equations, students are encouraged to analogize and obtain preliminary research methods for solving application problems with linear equations in two variables, leading them to learn how to use mathematical thinking methods to think and solve problems.

2.3 Differentiate understanding and consolidate cognition

Exploration 2: The vaccine transport vehicle in shot 2 departs from Beijing Kexing Pharmaceutical Co., Ltd. If driven at a speed of 80 kilometers per hour, it can transport the vaccine to A city after several hours. If the speed increases by 5 kilometers per hour, then in the same amount of time, the transport vehicle can reach B city 45 kilometers away from A. What is the time it takes for the transport vehicle to travel from Beijing Kexing to A city before accelerating? What is the distance to A city? (Show moving pictures while reading questions)

Teacher's guidance: Through the previous question, we have learned the steps to solve the application problem of a system of equations. Let's talk about it together (student's answer: "审设列解验答"). Everyone easily formed a solution idea for this question.

Expansion: In this problem, we can combine known and unknown quantities and use the method of drawing diagrams to help solve the problem. What is the clear known quantity? What about the unknown quantity (speed)? Is this question (distance and time) well-known to us? What is the basic formula used to solve this type of problem? (Speed × Time = distance). This is also the key to the problem-solving process: **finding equal quantity relationships**.

Joint activities between teachers and students：Students independently think and complete problem-solving, and teachers provide feedback and summaries.

Design intention: To understand the method of solving application problems using a system of equations from the perspectives of assignment problems and itinerary problems, deepen the penetration of mathematical modeling ideas, and develop students' mathematical abstraction literacy Standardize problem-solving steps to help students understand the key to solving problems that are inseparable from their roots: finding equal relationships and highlighting teaching priorities.

2.4 Thinking and communication, flexible application

Exploration 3: In shot 3, the vaccine has been successfully transported to City A and vaccination work has begun. As a vaccination site in City A, Red Star Community receives vaccines in batches. The number of vaccinations in different batches varies, including 2 doses of inactivated vaccine and 3 doses of recombinant protein vaccine. As of the end of June, a total of 968 people have been vaccinated in Red Star Community, totaling 2479 doses. So, how many residents have received inactivated vaccine and recombinant protein vaccine respectively?

Joint activities between teachers and students：Students collaborate and exchange ideas in groups. Teachers invite group representatives to present on the blackboard, and teachers and students work together to provide feedback and improvement.

Exploration 4: In shot 4, China plans to send two specialized planes to transport a batch of vaccines to a certain country. If the first plane flies 300km before the second plane departs, then after 3 hours, both planes can arrive at the destination at the same time; If the first aircraft flies for 15 minutes, then the second aircraft can catch up with the first aircraft after flying for 2 hours. How many kilometers do the two aircraft fly per hour?

Joint activities between teachers and students：Students complete independently, the teacher randomly selects one student's answer to display, and teachers and students jointly evaluate and improve.

Design intention: Presenting mathematical models for allocation and scheduling problems again, deepening and consolidating understanding, developing mathematical modeling skills, and developing mathematical operation skills through independent completion of complete problem-solving.

Classroom exercise (textbook example 2):

Design intention: Through classroom exercises (textbook example 2), students are encouraged to understand the advantages of using the method of solving a set of equations through solving multiple problems, and to further develop the mathematical core literacy of expressing real problems with mathematical language and thinking about real problems with mathematical thinking.

2.5 Reflection summary, deepening and improving

Questions: What did you learn in this class? What problem has been solved? What ideas and methods have you mastered?

Joint activities between teachers and students：Teachers and students jointly summarized and summarized: not only did they understand how vaccines appeared and played a role in our lives, but they also explored the path of vaccines by forming ideas and standardized steps. They also conducted variant training and challenged themselves. They summarized and summarized the "chicken rabbit cage" model with multiple solutions to one problem, and felt that using equations to solve practical problems has simplified problems in daily life; Learned how to use diagrams to guide equations to solve application problems related to stroke; The commonly used problem-solving strategies/steps for solving application problems include "review, design, list, solve, verify, and answer". Utilize allocation problems and itinerary problem models to cultivate mathematical concepts such as transformation and equations, and develop core competencies such as mathematical modeling literacy.

Design intention: In addition to summarizing knowledge, the classroom summary should also summarize the learning process and research path of the entire class from a methodological perspective, so that students can deeply understand the mathematical ideas and develop their mathematical core literacy.

3 Teaching Reflection

3.1 Clarify research methods to reveal the essence of knowledge

In order to scientifically construct a mathematical knowledge system, the primary task is to help students clarify the research object and research methods. The research object of this lesson's problem-solving teaching is application problems, and the research method is to use equations as a bridge to communicate real problems and mathematical problems through the steps of "review, design, list, solve, verify, and answer". In summary, the essence of solving problems in this lesson is to "observe the real world with a mathematical perspective, think about the real world with mathematical thinking, and express the real world with mathematical language."

3.2 Optimizing design to improve efficiency

In this section of teaching, the author uses a camera lens as a medium and connects the knowledge points of the entire class through the production, transportation, vaccination, and foreign aid as the main thread, further reflecting the

coherence of knowledge. For example 2 in the textbook, it is moved back to the position of expansion training as a supplement, guiding students to understand the principle of "different paths lead to the same goal" through multiple solutions to a single question, while highlighting the importance of the knowledge learned in this section, The computational complexity is small, the superiority is obvious, and it fully reflects the mathematical thinking of mathematical modeling. The teaching design of the entire class is reasonable and orderly, and through moderate "secondary development" of textbooks, the teaching design is reasonably optimized, the teaching plan is adjusted, and classroom efficiency is improved

3.3 Infiltration of ideological development literacy

In classroom teaching, guiding students to study problem-solving ideas through the use of examples and analogies, training students to consolidate problem-solving methods through the infiltration and transformation of typical example problems, and selecting problem-solving methods through classroom exercises. By analyzing multiple solutions to a single problem, students are guided to "reveal hidden information and extract information" from multiple perspectives and "flexibly transform and translate information", "From known to unknown, communicate and build bridges" [4], conduct basic analysis of problem-solving ideas and process analysis of problem-solving methods, enable students to think carefully, make correct judgments and quickly answer, cultivate students' logical and agile thinking, and then comprehensively develop and improve their core competencies in mathematical modeling, mathematical abstraction, mathematical operations, etc.

4 Conclusion

Based on core literacy teaching, teachers are required to clarify the cultivation elements of literacy, design teaching plans reasonably, and enable students to form and develop mathematical core literacy while mastering knowledge and skills. Therefore, when preparing for classes before class, teachers should study the new curriculum standards seriously, concentrate on reading new textbooks, carefully design each class, and form a scientific and reasonable teaching plan; In classroom teaching, teachers need to create appropriate teaching situations, propose appropriate mathematical problems, design reasonable student activities, and optimize the teaching process to ensure the accurate implementation of mathematical core competencies in classroom teaching!

REFERENCES

[1]. Edited by CaoYiming. (2020). Mathematics Teaching Theory. Beijing: Higher Education Press. (pp. 143).

[2]. Ministry of Education of the People's Republic of China. (2022).Mathematics Curriculum Standards for Compulsory Education (2022 Edition). Beijing: Beijing Normal University Press. (pp. 5-11).

[3].Edited by Zhan Tao. (2012).Mathematics in Compulsory Education Textbook (Grade 7, Volume 2). Qingdao: Qingdao Publishing House. (pp. 61-65).

[4].Wang Dicheng. (2013). *The important aspect of problem-solving teaching is to teach students analytical methods.* Research on Mathematics Teaching. 2013(10),(pp.9-13).

[5].Li Shuling. (2023). *Research on Teaching Strategies for Middle School Mathematics Problem Solving from the Perspective of Core Literacy.* Friends of Mathematics. 2023(11),(pp.29-31).

[6].Zhang Shuangxi. (2023). *The application of transformation thinking in middle school mathematics problem-solving teaching.* Research on Mathematical, Physical and Chemical Problem Solving. 2023(11),(pp.11-13).

[7].Li Qingquan. (2021). *An Analysis of the Training Strategies for Middle School Mathematics Problem Solving Ability.* Mathematics Teaching Communication. 2021(03),(pp.80-81+88).

[8].Sun Fei. (2020). *Exploration of Strategies for Solving Middle School Mathematics Problems.* Research on Mathematical, Physical and Chemical Problem Solving. 2020(08),(pp.15-16).

[9].Ren Sixue. (2020). *Analysis of teaching strategies and problem-solving skills for middle school mathematics application problems.* Exam Weekly. 2020(55),(pp.98).

[10].Li Shaojun. (2013). *Effectively Breaking Through the Difficulties of Teaching Applied Problems in Solving Systems of Equations.* Teaching Research. 2013(04),(pp.75).