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Case study Based on Ogane's Problem-Solving Process

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Abstract: In the 1980s, there were problems in the teaching of problem-solving in mathematics in the former Soviet Union, such as formulaic solutions and a lack of emphasis on teaching mathematical thinking. Therefore, Oganesian believes that in the traditional education system and in a large number of mathematical teaching practices, the role of mathematical exercises in developing students' mathematical level is limited. Mathematical exercises are difficult to test students' other aspects of mathematical development and ideological education factors. Introducing new teaching models is an urgent need for teaching mathematical problem-solving. Based on this, Oganesian proposed four steps for solving problems in his book "Teaching Methods for Primary and Secondary School Mathematics": understanding the problem conditions-developing a solution plan - implementing the solution plan - researching the obtained solutions. The article takes the "theorem for determining the perpendicularity of a straight line to a plane" in mathematics as an example, and analyzes its application based on the process of solving problems using the Oganesian method. By introducing mathematical problems into the Oganesian phase problem-solving process, gradually solving problems, delving into the vertical relationship between lines and planes, and analyzing their significance and role in practical applications. Assist students in cultivating thinking methods and problem-solving strategies for mathematical problem-solving systems, guide them to solve mathematical problems more deeply and accurately, enhance the problem-solving ability of mathematical learners, and optimize their thinking logic.
Keywords: Oganesian; problem-solving research; middle school mathematics; theorem of perpendicularity between a line and a plane

Research background

In 1957, the Soviet Union launched the first artificial Earth satellite, and the United States was shocked to realize the backwardness of its mathematics education. In 1961, the National Committee of American Mathematics Teachers published a document called "The Revolution of School Mathematics", which was widely known as the "New Mathematical Movement". Around 1970, the emphasis on fundamentals became the mainstream of mathematics education at that time, and the traditional view that "only through repeated and extensive practice can students better master basic knowledge and skills" regained its dominance[1]. Later, with the rapid development of science and technology, the information age quietly arrived, and the demand for high-end mathematical talents gave rise to people proposing that "mathematics should adapt to the trend of the times." Therefore, since the 1980s, problem solving has become the trend of international mathematics education. In its Agenda for Action (1980), the American Council of Mathematics Teachers explicitly stated that "problem-solving must be the core of high school mathematics education in the 1980s," "mathematics curriculum should be organized around problem-solving," and "mathematics teachers should create a classroom environment that enables problem-solving to flourish." At the 6th International Mathematical Education Conference held in 1988, "problem-solving, modeling, and application" were emphasized As one of the seven major research topics, it has been highly valued by mathematicians from various countries[2]. At recent global conferences of modern mathematics teachers, problem-solving has always been the main topic.

The Curriculum Standards for Ordinary High Schools (2017 Edition, 2020 Revision) mention that through the study of high school mathematics courses, students can acquire the necessary mathematical basic knowledge, basic skills, basic ideas, and basic activity experience (referred to as the "Four Basics") for further learning and future development, and improve their ability to discover and propose problems from a mathematical perspective, analyze and solve problems (referred to as the "Four Abilities")[3]. The improvement of problem-solving ability depends on the training of problem-solving. Without necessary training, students will find it difficult to achieve a deep grasp of the knowledge points they have learned, and thus cannot develop basic problem-solving skills. Therefore, mathematics teaching should be carried out in the form of problem-solving. Teachers should present the knowledge points of the textbook to students in the form of problems in teaching, so that students can master knowledge, develop intelligence, cultivate skills, and form thinking in the thinking activities of seeking, exploring, and solving problems. However, currently in the teaching of problem-solving, some teachers overly emphasize skills or problem-solving routines, without focusing on the general process of problem-solving, without reflecting the refinement and generation of the problem-solving process, and without letting students understand the general process and general ideas of problem-solving[4]. Therefore, research should be conducted on how to more effectively carry out problem-solving teaching.

In order to consolidate the knowledge and methods learned by students, experience the mathematical ideas hidden behind mathematical knowledge and problem-solving methods, and effectively cultivate mathematical abilities and the ability to learn mathematics, it is necessary to conduct reasonable problem-solving exercises. Soviet mathematics educator

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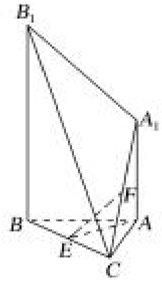
Oganesian pointed out in his "Teaching Methods for Primary and Secondary School Mathematics" [5] that "it is necessary to pay attention to the potential of many exercises to further expand their mathematical, developmental, and educational functions." Oganesian conducted research on solving mathematical problems and divided the problem-solving process into four stages: "understanding problem conditions, formulating problem-solving plans, implementing problem-solving plans, and researching the solutions obtained." [6]The article will analyze and explain the process of solving the Oganesian phase problem using the problem of "the theorem for determining the perpendicularity of a straight line to a plane" as an example.

Case Analysis

Design exercise questions with the purpose of determining the perpendicularity of a straight line to a plane, as follows:

As shown in the figure, it is known that the $AA_1\perp$ plane ABC, $BB_1 /\!/ AA_1$，$AB=AC=3$，$BC=2\sqrt{5}$，$AA_1=\sqrt{7}$，$BB_1=2\sqrt{7}$，and points E and F are the midpoints of BC and A_1C



(1) Verification: EF // plane A_1B_1BA;

(2) Find the size of the angle between line A_1B_1 and plane BCB_1

[Analysis] This question tests the application of the judgment theorem for line plane parallelism and line plane verticality, as well as the proof and solution of line plane angles. The two questions are closely related and easily accepted by students, providing them with development space to help them consolidate their knowledge and acquire problem-solving skills.

(1) Connect A_1B, determine $EF /\!/ A_1B$ from the meaning of the question, and then draw a conclusion from the criterion of parallel lines and surfaces;

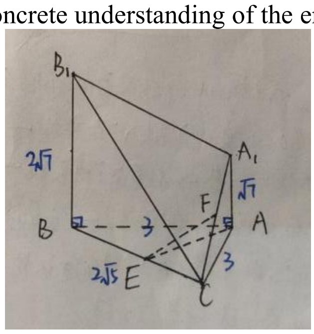
(2) Firstly, by combining the conditions in the question and using the judgment theorem of line plane perpendicularity, it can be directly proven that the straight line $AE\perp$ plane BCB_1; Secondly, take the midpoints M and N of B_1B and B_1C respectively, connect A_1M and A_1N, and use the relevant known conditions in the question to prove $A_1N /\!/ AE$. Because the line $AE\perp$ plane BCB_1 can be obtained, then the $A_1N\perp$ plane BCB_1 can be obtained, and $\angle A_1B_1N$ is the angle formed by the line A_1B_1 and the plane BCB; Combined with the quantity relationship in the question, $A_1B_1=4$ and $A_1N=2$ can be obtained. Finally, it can be concluded that in $Rt_{\triangle}A_1B_1N$, $\sin\angle A_1B_1N=\frac{A_1N}{A_1B_1}=\frac{1}{2}$, then $\angle A_1B_1N=30°$

Problem solving process

1. Understanding the conditions of the problem

During the process of reading questions, students should recognize the conditions and requirements given to us in the question stem, deeply understand and analyze the various elements in the conditions (or elements in the target), use their mathematical knowledge network they usually build, search for corresponding information, and establish a connection between the conditions and conclusions of the problem and existing knowledge and experience.

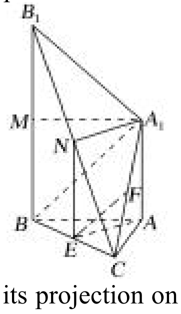
①When starting to study the conditions of a problem, it is necessary to compare them with intuitive graphics and label the known conditions in the graph to help think about the problem. Accurately representing the conditions of a problem in graphics means having a clear, explicit, and concrete understanding of the entire context of the problem.



② Clearly understand the various elements in the context of the problem. Be sure to clarify which elements are known; What is unknown. This requires deriving implicit conditions based on the conditions given in the question, establishing

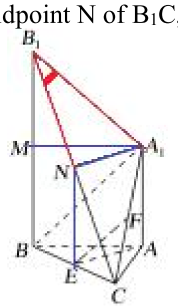
Due to the $AA_1\perp$ plane ABC, according to the definition of line plane perpendicularity, AA_1 is perpendicular to any straight line in plane ABC; Due to $BB_1 /\!/ AA_1$, according to the inverse theorem of the line plane perpendicular property theorem, it can be obtained that BB_1 is perpendicular to any straight line in plane ABC; Due to AB=AC and point E being

the midpoint of BC, so AE⊥BC.
The conclusion to be proved in question (1) is that the EF∥ plane A_1B_1BA needs to find a straight line parallel to EF in plane A_1B_1BA according to the theorem of line plane parallelism.
Question (2): To find the projection of oblique line A_1B_1 on plane BCB_1, it is necessary to first find the perpendicular line passing through point A_1 on plane BCB_1.
③ Thoroughly contemplate the meaning of each word (symbol, term) in the problem description; Try to identify the important elements of the problem and use visual symbols to mark known and unknown elements on the graph. Try adding auxiliary lines on the diagram or sketch.
Question (1): Find a straight line parallel to EF in plane A_1B_1BA. Due to the fact that points E and F are both midpoints, they are important elements. If there are two midpoints, one can try to solve the parallel problem using the knowledge of triangle median lines.
Question (2): Find the perpendicular line passing through point A_1 on plane BCB_1. Analyzing the known conditions, due to AE⊥BC, AE⊥BB_1, and BC∩BB_1=B, according to the criterion of line plane perpendicularity, it can be concluded that the AE⊥ plane BCB_1. Can we draw a straight line parallel to AE through point A_1.
④ Try to understand the conditions of the problem as a whole, identify its characteristics, and think about whether you have encountered any similar problems to this problem before.
Question (2): When creating a straight line parallel to a known line, it is generally necessary to construct a median line of a triangle or a parallelogram.
⑤ Think carefully about whether the description of the problem can be interpreted differently, whether there are any unnecessary or contradictory things in the conditions of the problem, and whether there are still any missing conditions.
The first question is to transform the problem and express it in another mathematical language. To prove that EF is parallel to a straight line in plane A_1B_1BA, the missing condition is EF∥l (where l is a straight line in plane A_1B_1BA). Therefore, when solving the problem, it is necessary to determine a straight line parallel to EF in plane A_1B_1BA.
Question (2): To determine the angle between a straight line and a plane, the missing condition is a visual representation of the angle formed. In order to visually represent this angle, according to the definition of the angle between the line and the plane, draw a perpendicular line from a point on the line to the plane to obtain the perpendicular foot, and then connect the perpendicular foot with the intersection point of the line and plane.
⑥ If it is possible to use a general mathematical method that you are familiar with when solving problems (such as equations, geometric transformations, coordinate or vector methods, etc.), try to use the language of that method to represent the elements of the problem as much as possible.
2.Develop a problem-solving plan
(1) Verification: EF∥ plane A_1B_1BA;
The ultimate goal of this question is to prove that the line EF∥ plane A_1B_1BA is parallel to the plane. According to the criterion of parallelism between lines and planes (if a line outside the plane is parallel to a line inside the plane, then the line is parallel to the plane), the parallelism between lines can be used to determine that the line is parallel to the plane. This is a common method for handling spatial positional relationships, which converts the parallelism between lines and planes (spatial problems) into parallelism between lines (planar problems). The key to proving that a line is parallel to a plane is to find a line that is parallel to a line in the plane. Therefore, to prove the plane A_1B_1BA of EF, it is necessary to find a straight line parallel to EF within the plane A_1B_1BA.
According to the known conditions, points E and F are the midpoints of BC and A_1C, respectively. If a straight line can be found in plane A_1B_1BA to form a triangle with BC and A_1C and EF is exactly the midpoint of the triangle, then EF can be made parallel to the line being found. It is obvious that EF is the median line of triangle CA_1B.
Therefore, the solution plan for this problem is to connect AB, combined with the triangle median line theorem, to obtain EF parallel to A_1B, and combined with the judgment theorem that a straight line is parallel to a plane.
(2) Find the size of the angle between line A_1B_1 and plane BCB_1



The angle formed by a diagonal line on a plane and its projection on the plane is called the angle formed by this line and the plane. This question tests the method of finding the perpendicular angle between a straight line and a plane, as well as the angle between a straight line and a plane. The key to finding the angle between a straight line and a plane is to find the line that is perpendicular to the plane, establish the angle, and solve the triangle
Firstly, to find the projection of oblique line A_1B_1 on plane BCB_1, it is necessary to first find the perpendicular line passing through point A_1 on plane BCB_1. There are two solutions to proving that a straight line is perpendicular to a plane. The first one is the definition of a line plane being perpendicular (generally, if the straight line l is perpendicular to any

line in the plane α, we say that line l is perpendicular to the plane α each other, denoted as $l \perp \alpha$); Another criterion is the theorem for determining the perpendicularity of a line and a plane (if a line is perpendicular to two intersecting lines in a plane, then the line is perpendicular to that plane). The theorem for determining the verticality of a line and plane reflects the mutual transformation between "a line is perpendicular to a plane" and "a line is perpendicular to a line".
Let's first assume that the perpendicular foot of A_1 on plane BCB_1 is on B_1B, because B_1B is twice that of A_1A, we can try to obtain the midpoint M. At this point, it is necessary to verify whether A_1M is perpendicular to plane BCB_1. If the first method and the definition of line plane perpendicularity are used, it is not realistic to prove that A_1B_1 is perpendicular to every straight line in plane BCB_1. But if a straight line that is not perpendicular to A_1M can be found in plane BCB_1, it can prove that A_1B_1 is not perpendicular to plane BCB_1. If we choose the second method, which is the theorem for determining the perpendicularity of lines and surfaces, we only need to find two intersecting lines perpendicular to A_1B_1 in plane BCB_1 to prove it.
Let's first use the first method to verify: $A_1M \parallel AB$. Based on the known conditions in the question, it can be determined that $\triangle ABC$ is an isosceles triangle, so $\angle ABC \neq 90°$, and AB is not perpendicular to BC, so A_1M is also not perpendicular to BC. At this point, we find a straight line BC in plane BCB_1 that is not perpendicular to A_1M, so it is not a perpendicular line to plane BCB_1. Because it has been known that A_1M is not perpendicular to the plane BCB_1, the second method does not need to be used.
Let's assume that the perpendicular foot of A_1 on plane BCB_1 is on B_1C, and we can also obtain the midpoint N. Next, we will verify whether A_1N is the perpendicular line of plane BCB_1. Similar to the previous hypothesis, there are two verification methods. We found that when using the first method to find a straight line in plane BCB_1 that is not perpendicular to A_1N, it repeatedly collides with the wall, all parallel to A_1N. Therefore, we use another method to find two intersecting lines perpendicular to A_1B_1 in plane BCB_1. Since AB=AC, taking the midpoint E of BC, gives $AE \perp BC$. It seems easier to find a straight line perpendicular to AE, which is parallel to A_1N. Therefore, we can convert the validation of $A_1N \perp$ plane BCB_1 to $AE \perp$ plane BCB_1. So we can find a straight line perpendicular to AE in plane BCB_1. By analyzing the known conditions $AA_1 \perp$ plane ABC, $BB_1 \parallel AA_1$ can be obtained as $BB_1 \perp$ plane ABC. Then, according to the property theorem of line plane perpendicularity, $AE \perp BB_1$ can be obtained, and BC intersects with BB_1 at point B. Therefore, we have to prove that A_1N is the perpendicular line of plane BCB_1, N is the perpendicular foot, and $\angle A_1B_1N$ is the angle formed by the straight line A_1B_1 and plane BCB_1.
Finally, the angle between the straight line A_1B_1 and the plane BCB_1 can be solved based on the angle relationship of the triangle.
Therefore, the solution plan for this problem is to take the midpoint N of B_1C, prove that $A_1N \perp$ plane BCB_1, find the angle $\angle A_1B_1N$ formed by the line and plane, and solve the triangle.
3. Implement the problem-solving plan
Implementing a problem-solving plan involves putting all the details of the plan into practice, while also making amendments to the plan by comparing it with the known conditions and selected evidence; selecting a method for describing the solution process and writing the solution, writing the results, and so on.
(1) Proof: As shown in the figure, connect A_1B.
In $\triangle A_1BC$, since E and F are the midpoints of BC and A_1C, respectively, $EF \parallel BA_1$.
Implement the construction of the median line in a triangle
And because $EF \not\subset A_1B_1BA$, $A_1B \subset$ plane A_1B_1BA,
Therefore, $EF \parallel$ plane A_1B_1BA.
Using the theorem of parallelism between a straight line and a plane
(2) Solution: Take the midpoint M of BB_1 and the midpoint N of B_1C, and connect A_1M, A_1N, and NE.



Because AB=AC, E is the midpoint of BC,
So $AE \perp BC$.
Because $AA_1 \perp$ plane ABC, $BB_1 \parallel AA_1$,
Therefore, BB1 is perpendicular to the plane ABC,
and $AE \subset$ plane ABC,
Therefore, $BB_1 \perp AE$.
Apply the theorem of the perpendicularity of line and plane
And because $BC \cap BB_1$ is at point B, BC and BB_1 are contained in the plane BCB_1,
Therefore, $AE \perp$ plane BCB_1.

Apply the theorem of perpendicularity between line and plane
because N and E are the midpoints of B_1C and BC,
So $NE//B_1B$, $NE=\frac{1}{2}B_1B$,
Therefore, $NE//A_1A$ and $NE=A_1A$,
Therefore, $A1N//AE$ and $A_1N=AE$.
And because AE is perpendicular to the plane BCB_1,
Therefore, $A_1N\perp$ plane BCB_1,
Therefore, $\angle A_1B_1N$ is the angle between the straight line A_1B_1 and the plane BCB_1.
In $\triangle$ ABC, we can get AE=2
So $A_1N=AE=2$.
because $BM // AA_1$, $BM=AA_1$,
Therefore, the quadrangle $MBAA_1$ is a parallelogram,
Therefore, $A_1M//AB$, $A_1M=AB$,
From $AB\perp BB_1$, we have $A_1M\perp BB_1$.
In Rt $\triangle A_1MB_1$,
Then $A_1B_1=4$.
In Rt $\triangle A_1NB_1$,
$\frac{A_1N}{A_1B_1}=\sin\angle A_1B_1N=\frac{1}{2}$,
Therefore, $\angle A_1B_1N=30°$.
Therefore, the angle formed by the straight line A_1B_1 and the plane BCB_1 is 30°.
4. Solution obtained from the research
By combining the conditions identified in the diagram and visually perceiving the graph, it is possible to grasp the correctness of the result to a certain extent. Recall the solution idea, whether there are logical errors or overlooked points, and consider whether other learned knowledge can be used to solve this problem. For example, you can try to use the coordinate representation and operation of space vectors to solve this problem, which is to verify the correctness of the result obtained by the previous method through another solution.
Reflect on which striking known conditions can be associated and activated in the mind to activate existing mathematical knowledge, and then derive some slightly implicit conclusions or conditions, such as seeing the midpoint and thinking of the midline theorem, obtaining $A_1B//FE$, and further comprehensively applying the knowledge network (schema) of the existing mathematical content in the mind, that is, the network of related definitions, decision theorems, and property theorems to naturally solve the problem.
Combing the definitions, decision theorems, and property theorems used in the problem, and experiencing their application in specific problems, familiarize yourself with their specific usage methods. For example, in the first question, it is crucial to prove that a straight line is parallel to a given plane. No matter how the problem changes, this is the key point that needs to be firmly grasped. The second question requires the size of the angle formed by the line and the plane. Naturally, it is necessary to clearly recall the definition of the angle formed by the line and the plane in your mind. The key to solving the problem lies in constructing this angle, and the difficulty in constructing this angle lies in where the "foot" in the definition actually falls. This depends on students' clarity of understanding between the conditions of the problem, and through intuitive imagination, it is guessed that the foot may fall on the midpoint of B_1C. Finally, combining the conditions in the problem with relevant decision theorems and property theorems learned previously, verify the conjecture.

Results and Discussion
The study of Oganesian's problem-solving has a profound impact on mathematics education in primary and secondary schools. It not only enhances students' mathematical abilities and thinking qualities, but also promotes the innovation and development of mathematics teaching methods. It is of great significance in cultivating students' innovative spirit and practical ability in the following aspects.
1. Cultivate students' mathematical thinking ability
Mathematical thinking ability refers to the ability to understand and apply mathematical concepts and principles for logical reasoning and problem-solving. The Oganesian method of problem-solving guides students to start from specific problems, gradually abstract, and cultivate their ability to transform practical problems into mathematical models. This method helps students establish connections between mathematical concepts, understand the internal logic of mathematical knowledge, and thus enhance their mathematical thinking abilities. For example, by solving geometric problems, students can learn how to construct effective proofs, which is an important aspect of mathematical thinking.
2. Enhance students' innovation and problem-solving abilities
Oganesian's problem-solving research encourages students to adopt multiple approaches and methods to solve mathematical problems. This open problem-solving process encourages students to diverge their thinking and try innovative solutions. In this process, students not only learn standard problem-solving skills, but more importantly, they learn how to flexibly apply these skills and other knowledge to solve new and unknown problems. This ability is extremely valuable for a student's entire learning career and even their future career.
3. Enhance students' logical reasoning ability

In the Oganesian's problem-solving method, logical reasoning is the core of the problem-solving process. Students are required not only to find solutions to problems, but also to clearly explain their problem-solving strategies, including using appropriate mathematical language and logical reasoning to prove that their answers are correct. This type of training helps students develop rigorous thinking habits and improve their logical reasoning abilities.

4. Improve students' interest and confidence in learning

By solving challenging mathematical problems, students can experience the joy of solving problems and the satisfaction of success. The problems in Oganesian's problem-solving research are often designed to be both interesting and inspiring, which can stimulate students' curiosity and exploratory desire. When students find solutions to problems through their own efforts, their confidence will be greatly enhanced, and this positive learning attitude is crucial for their long-term development.

5. Promote innovation and development of mathematical teaching methods

The research on Oganesian's problem-solving is not only beneficial to students, but also encourages teachers to reflect and innovate teaching methods. In order to better implement this teaching strategy, teachers need to constantly learn new teaching concepts and methods, and find or design mathematical problems that can stimulate students' interest and thinking. This continuous professional development helps to improve the quality of teaching, making mathematics education more vivid, effective, and in line with the actual needs of students.

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